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Numerical optimisation of superplastic deformation

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Based on an approach due to Padmanabhan and Davies, a multi-dimensional regression analysis has been developed which predicts the superplastic deformation parameters of m (the strain-rate sensitivity index) and K (the strength parameter) as functions of strain rate, grain size and temperature. Further analysis enables the optimisation of the operating conditions (for minimum power consumption) through a prediction of the external load and power consumption using the predicted values of m and K. The procedure has been validated by applying it for the analysis of the experimental data pertaining to the tin–lead eutectic alloy. It has been pointed out that the technique could be useful for problems (not necessarily in the area of superplasticity) where a particular parameter depends on a number of independent variables.

1. Introduction

"Micro-grained" or "structural" superplasticity is a well-established phenomenon [1–4]. Metallic materials possessing a stable, ultra-fine grain size (usually less than 10 μm) exhibit extremely large deformation of many hundreds of per cent when subjected to a small tensile load within a certain strain rate interval and a deformation temperature in excess of about $0.4T_m$ (where T_m is the melting point on the absolute scale). A number of commercial applications that make use of superplastic forming have already been identified [2, 5–8].

Isothermal superplastic deformation is often represented by an equation [1, 2]

$$\sigma = K\dot{\varepsilon}^m \tag{1}$$

where σ is the applied stress, $\dot{\varepsilon}$ is the strain rate, K (the strength parameter that equals the flow stress at a strain rate of unity) and m (the strain-rate sensitivity index) are material parameters that depend strongly on strain rate ($\dot{\varepsilon}$), grain size (L) and temperature (T). For superplastic flow, $0.3 < m < 1$ and the resistance to necking increases with m [1, 2].

On the other hand, contrary to experimental results many analyses of superplastic forming treat m and K as constants. But in a more realistic model the internal variations in m and K with $\dot{\varepsilon}$, L and T have to be taken into account, while calculating the forming load, power consumption etc. Then, reliable optimisation of the operating conditions will become possible. To this end, a numerical procedure based on multi-dimensional regression analysis is presented in this paper. This technique can be applied to other areas of research also.

2. Analysis and results

The basic approach was presented by Padmanabhan and Davies [9]. Although the procedure outlined was general, as an example these authors treated the three dimensional case when m and K were functions of $\dot{\varepsilon}$ and L. In this paper the four dimensional situation in which $m = f_1(\dot{\varepsilon}, L, T)$ and $K = f_2(\dot{\varepsilon}, L, T)$ is analysed (By induction, the technique can be generalised for an n-dimensional space, where a given dependent variable is influenced by $(n - 1)$ independent variables.)

The strain rate, measured in s^{-1}, the grain size, measured in μm, and the temperature in degrees Kelvin are transformed to variables X, Y, Z by the equations

$$X = -\log_{10}\dot{\varepsilon}$$
$$Y = L \tag{2}$$
$$Z = \left(\frac{T - 273}{100}\right)$$

The variables are thus limited to positive values of a similar order of magnitude and this will minimise the computational errors [10].

It is assumed that m can be represented by a polynomial of the form

$$m = \sum_{r,s,t=0}^{r+s+t=N} A_{rst}X^rY^sZ^t + \varepsilon \tag{3}$$

where A_{rst} are numerical coefficients and ε is a random error term. This is an equation of degree $N = r + s + t$. It is assumed that the standard deviation is the same for all observations and the random error term

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is set as zero. The predicted value of m is then given by

$$m_{\text{predicted}} = \sum_{r,s,t=0}^{r+s+t=N} A_{rst} X^r Y^s Z^t \quad (4)$$

The maximum possible degree of the prediction equation is limited by the number of experimental observations available. Assuming n experimental values for m, each value corresponding to a set of X $(= -\log_{10}\dot{\varepsilon})$, $Y(=L)$, $Z(=(T-273)/100)$ the coefficients A_{rst} in Equation 4 can be uniquely determined, if the number of coefficients is not greater than n. Data are required in the form of experimental m values for different sets of strain rate, grain size and temperature.

Let m_{ijk} denote the m value corresponding to a set of values $X = X_i$, $Y = Y_j$, $Z = Z_k$. Then, Equation 4 becomes

$$(m_{ijk})_{\text{predicted}} = \sum_{r,s,t=0}^{r+s+t=N} A_{rst} X_i^r Y_j^s Z_k^t \quad (5)$$

with the constraining conditions $r \leqslant C - 1$, $s \leqslant D - 1$, $t \leqslant E - 1$, $r + s + t \leqslant N$, where C is the number of observations of X for each Y–Z combination, D is the number of Y values for each Z value and E is the number of Z values available. A_{rst} are evaluated following the least squares procedure. The sum of the squares of the residuals is a function of the

TABLE I Experimental data for the tin–lead eutectic alloy [11]

Temperature (T-273)	L (μm)	$\dot{\varepsilon}$ (s^{-1})	'm'	σ (psi)*
26	7.5 ± 0.8	3.18×10^{-5}	0.31	2780
26	7.5 ± 0.8	7.70×10^{-5}	0.22	3550
26	7.5 ± 0.8	1.55×10^{-4}	0.19	4005
26	7.5 ± 0.8	2.85×10^{-4}	0.17	4530
26	7.5 ± 0.8	6.69×10^{-4}	0.15	5110
26	7.5 ± 0.8	1.31×10^{-3}	0.13	5780
26	7.5 ± 0.8	2.56×10^{-3}	0.11	6140
26	7.5 ± 0.8	6.37×10^{-3}	0.08	6730
60	7.5 ± 0.8	3.18×10^{-5}	0.24	1810
60	7.5 ± 0.8	7.70×10^{-5}	0.28	2310
60	7.5 ± 0.8	1.55×10^{-4}	0.25	2780
60	7.5 ± 0.8	2.85×10^{-4}	0.21	3140
60	7.5 ± 0.8	6.69×10^{-4}	0.14	3770
60	7.5 ± 0.8	1.31×10^{-3}	0.11	4005
60	7.5 ± 0.8	2.56×10^{-3}	0.08	4260
60	7.5 ± 0.8	6.37×10^{-3}	0.07	4530
120	7.5 ± 0.8	3.28×10^{-5}	0.60	570
120	7.5 ± 0.8	7.93×10^{-5}	0.44	870
120	7.5 ± 0.8	1.55×10^{-4}	0.37	1180
120	7.5 ± 0.8	3.03×10^{-4}	0.31	1510
120	7.5 ± 0.8	7.11×10^{-4}	0.26	1810
120	7.5 ± 0.8	1.39×10^{-3}	0.23	2180
120	7.5 ± 0.8	2.72×10^{-3}	0.20	2610
120	7.5 ± 0.8	6.37×10^{-3}	0.15	2950
170	7.5 ± 0.8	3.18×10^{-5}	0.58	60
170	7.5 ± 0.8	7.93×10^{-5}	0.58	90
170	7.5 ± 0.8	1.55×10^{-4}	0.60	140
170	7.5 ± 0.8	2.85×10^{-4}	0.62	210
170	7.5 ± 0.8	7.11×10^{-4}	0.61	350
170	7.5 ± 0.8	1.39×10^{-3}	0.59	540
170	7.5 ± 0.8	2.56×10^{-3}	0.54	730
170	7.5 ± 0.8	9.47×10^{-3}	0.37	1420
26	5.5 ± 0.6	3.18×10^{-5}	0.31	2410
26	5.5 ± 0.6	7.46×10^{-5}	0.27	3070
26	5.5 ± 0.6	1.55×10^{-4}	0.24	3680
26	5.5 ± 0.6	3.03×10^{-4}	0.21	4410
26	5.5 ± 0.6	7.11×10^{-4}	0.17	4980
26	5.5 ± 0.6	1.39×10^{-3}	0.14	5620
26	5.5 ± 0.6	2.56×10^{-3}	0.12	5970
26	5.5 ± 0.6	5.99×10^{-3}	0.10	6740
60	5.5 ± 0.6	3.18×10^{-5}	0.35	1160
60	5.5 ± 0.6	8.18×10^{-5}	0.40	1670
60	5.5 ± 0.6	1.55×10^{-4}	0.36	2130
60	5.5 ± 0.6	3.03×10^{-4}	0.29	2720
60	5.5 ± 0.6	7.11×10^{-4}	0.21	3260
60	5.5 ± 0.6	1.31×10^{-3}	0.17	3680
60	5.5 ± 0.6	2.48×10^{-3}	0.15	4150
60	5.5 ± 0.6	5.99×10^{-3}	0.13	4690
120	5.5 ± 0.6	3.18×10^{-5}	0.26	200

TABLE I (*continued*)

Temperature (T-273)	L (μm)	$\dot{\varepsilon}$ (s^{-1})	'm'	σ (psi)*
120	5.5 ± 0.6	7.46×10^{-5}	0.49	290
120	5.5 ± 0.6	1.46×10^{-4}	0.65	390
120	5.5 ± 0.6	2.68×10^{-4}	0.54	560
120	5.5 ± 0.6	6.49×10^{-4}	0.46	860
120	5.5 ± 0.6	1.16×10^{-3}	0.39	1160
120	5.5 ± 0.6	2.26×10^{-3}	0.30	1400
120	5.5 ± 0.6	4.99×10^{-3}	0.22	1670
170	5.5 ± 0.6	7.93×10^{-5}	0.45	70
170	5.5 ± 0.6	1.55×10^{-4}	0.58	100
170	5.5 ± 0.6	2.85×10^{-4}	0.63	150
170	5.5 ± 0.6	7.11×10^{-4}	0.62	260
170	5.5 ± 0.6	1.39×10^{-3}	0.58	420
170	5.5 ± 0.6	2.72×10^{-3}	0.53	560
170	5.5 ± 0.6	6.37×10^{-3}	0.46	860
170	5.5 ± 0.6	1.17×10^{-2}	0.38	1160
26	2.0 ± 0.2	2.99×10^{-5}	0.29	390
26	2.0 ± 0.2	7.46×10^{-5}	0.41	530
26	2.0 ± 0.2	1.37×10^{-4}	0.47	710
26	2.0 ± 0.2	2.77×10^{-4}	0.50	970
26	2.0 ± 0.2	6.29×10^{-4}	0.51	1490
26	2.0 ± 0.2	1.31×10^{-3}	0.48	2140
26	2.0 ± 0.2	2.33×10^{-3}	0.44	2910
26	2.0 ± 0.2	4.70×10^{-3}	0.37	3710
68	2.0 ± 0.2	3.18×10^{-5}	0.37	180
68	2.0 ± 0.2	7.70×10^{-5}	0.43	250
68	2.0 ± 0.2	1.46×10^{-4}	0.46	320
68	2.0 ± 0.2	2.85×10^{-4}	0.49	470
68	2.0 ± 0.2	6.90×10^{-4}	0.51	710
68	2.0 ± 0.2	1.35×10^{-3}	0.51	970
68	2.0 ± 0.2	2.48×10^{-3}	0.50	1400
68	2.0 ± 0.2	5.64×10^{-3}	0.48	2020
90	2.0 ± 0.2	3.18×10^{-5}	0.44	110
90	2.0 ± 0.2	7.70×10^{-5}	0.43	170
90	2.0 ± 0.2	1.46×10^{-4}	0.45	220
90	2.0 ± 0.2	2.85×10^{-4}	0.47	300
90	2.0 ± 0.2	6.90×10^{-4}	0.50	470
90	2.0 ± 0.2	1.35×10^{-3}	0.52	630
90	2.0 ± 0.2	2.48×10^{-3}	0.52	910
90	2.0 ± 0.2	5.47×10^{-3}	0.51	1320
120	2.0 ± 0.2	3.09×10^{-5}	0.45	55
120	2.0 ± 0.2	7.46×10^{-5}	0.51	85
120	2.0 ± 0.2	1.46×10^{-4}	0.52	120
120	2.0 ± 0.2	2.68×10^{-4}	0.53	170
120	2.0 ± 0.2	6.69×10^{-4}	0.53	250
120	2.0 ± 0.2	1.23×10^{-3}	0.55	390
120	2.0 ± 0.2	2.40×10^{-3}	0.57	560
120	2.0 ± 0.2	5.47×10^{-3}	0.59	860

*Zehr and Backofen [11] have reported the stress in psi.
1 psi = 6.8948 kPa.

coefficients A_{rst}, i.e.

$$\sum_{k=1}^{k=E}\sum_{j=1}^{j=D}\sum_{i=1}^{i=C}[(m_{ijk})_{\text{observed}} - (m_{ijk})_{\text{predicted}}]^2 = \phi(A_{000}, A_{001}, \ldots, A_{rst}) \qquad (6)$$

where $(m_{ijk})_{\text{observed}}$ is the experimental value of m_{ijk} and the corresponding $(m_{ijk})_{\text{predicted}}$ value is calculated using Equation (5).

The polynomial of interest was evaluated by minimising the function ϕ in Equation 6 by successively differentiating with respect to the coefficients and equating the first derivatives to zero. Thus in the case of a polynomial of the maximum possible degree, one has $(r + Cs + CDt)$ equations, each involving the summation of $(r + Cs + CDt)$ individual terms. Solving these equations, the $(r + Cs + CDt)$ coefficients A_{rst} are determined.

Using the above procedure, the following set of equations are derived.

$$\sum_{r,s,t=0}^{r+s+t=N} \underline{\alpha}_{(p+Cq+CDu)(r+Cs+CDt)}A_{rst} = \underline{\beta}_{(p+Cq+CDu)} \qquad (7)$$

where the general terms

$$\underline{\alpha}_{(p+Cq+CDu)(r'+Cs'+CDt')} = \sum_{k=1}^{k=E}\sum_{j=1}^{j=D}\sum_{i=1}^{i=C} X_i^{p+r'}Y_j^{q+s'}Z_k^{u+t'}$$

and

$$\underline{\beta}_{(p+Cq+CDu)} = \sum_{k=1}^{k=E}\sum_{j=1}^{j=D}\sum_{i=1}^{i=C}(m_{ijk})_{\text{observed}}X_i^pY_j^qZ_k^u.$$

In these expressions, p, q, u and r', s' and t' take on values from zero to $(p + Cq + CDu) = (r + Cs + CDt)$ and $(r' + Cs' + CDt') = (r + Cs + CDt)$ with $p \leqslant C - 1$, $r' \leqslant C - 1$, $q \leqslant D - 1$, $s' \leqslant D - 1$, $u \leqslant E - 1$ and $t' \leqslant E - 1$. From the $(r + Cs + CDt)$ equations, using Gaussian elimination one can determine the $r + Cs + CDt$ coefficients precisely.

The above procedure was also used for evaluating K as $F_2(X, Y, Z)$ using Equation 1 for estimating the experimental values of K from the experimental values of σ, $\dot{\varepsilon}$ and m.

The analysis was tested using the experimental data of Zehr and Backofen [11], pertaining to a tin–lead eutectic alloy (Table I). Polynomials of degrees up to the twelfth were determined for $m = F_1(X, Y, Z)$, the limit on the maximum degree of the polynomial being set by the number of experimental data points available. The individual powers of X, Y and Z were restricted to the seventh, third and second degrees respectively, again based on the availability of experimental data.

The number of coefficients to be evaluated for the polynomials of degrees one to twelve were 4, 10, 19, 30, 42, 54, 66, 77, 86, 92, 95 and 96. As noted by Padmanabhan and Davies [9], judgment is involved in the selection of the "best fit" polynomial, as a number of procedures are available [10]. Here the forward selection procedure was used, in which it is checked if on moving from a given degree of polynomial to the next there is a significant reduction in the sum of the squares of the residuals. That degree of the polynomial moving to which the reduction in the sum of the squares of the residuals was maximum, was selected as the "best fit" Equation 10. (One could have also selected the equation of the lowest degree which fell within a prescribed error range, this error being taken to be of the order of the accuracy of the experimental data [12]. But the errors in the measurements of Zehr and Backofen [11] are not known.)

TABLE II Selection of "best fit" polynomial for predicting the values of m

Degree of equation	Root mean square error (%)	Residual sum of squares	$\frac{(Er_n{}^{\dagger} - Er_{n+1}{}^{\ddagger}) \cdot 100^*}{Er_n}$ (%)
1	22.81	7.10×10^{-1}	–
2	14.11	2.72×10^{-1}	61.69
3	11.89	1.93×10^{-1}	29.04
4	9.32	1.18×10^{-1}	38.86
5	5.38	3.95×10^{-2}	66.53
6	2.61	9.30×10^{-3}	76.46
7	1.10	1.66×10^{-3}	82.15
8	0.13	2.12×10^{-5}	98.72
9	0.04	2.08×10^{-6}	90.19
10	0.05	3.36×10^{-6}	−61.54
11	0.01	1.28×10^{-7}	96.19
12	0.01	9.63×10^{-8}	24.77

*As the improvement in fit is due to the inclusion of the $(n + 1)$th degree, the computed value of the expression is assigned to the polynomial of degree $(n + 1)$.
† Er_n = Residual sum of squares of polynomial of degree n.
‡ Er_{n+1} = Residual sum of squares of polynomial of degree $(n + 1)$.

To evaluate the error in the prediction of the polynomials of different degrees the root mean square error, RMS error, was calculated as

$$\text{RMS error} = (1/\bar{m}_{\text{observed}}) \times \left[(1/n)\sum_{i=1}^{i=n}[(m_i)_{\text{observed}} - (m_i)_{\text{predicted}}]^2\right]^{1/2}$$
$$= (1/\bar{m}_{\text{observed}}) \times \left[(1/n)\sum_{k=1}^{k=E}\sum_{j=1}^{j=D}\sum_{i=1}^{i=C}[(m_{ijk})_{\text{observed}} - (m_{ijk})_{\text{predicted}}]^2\right]^{1/2} \qquad (8)$$

where $n = CDE$ = number of observations.

Table II displays the RMS error evaluated for the different degrees of the polynomials. Using the forward selection procedure, the "best fit" equation for m was decided as the polynomial of degree eight.

Thus, for the tin–lead eutectic alloy the "best fit" polynomial for calculating m was chosen as

$$(m)_{\text{predicted}} = \sum_{r,s,t=0}^{r+s+t=8} A_{rst}X^rY^sZ^t \qquad (9)$$

where

$A_{000} = +2.9334 \times 10^1$ $\quad A_{100} = -6.8185 \times 10^1$ $\quad A_{200} = +6.0813 \times 10^1$
$A_{300} = -2.7897 \times 10^1$ $\quad A_{400} = +7.3573$ $\quad A_{500} = -1.1453$
$A_{600} = +9.8830 \times 10^{-2}$ $\quad A_{700} = -3.6094 \times 10^{-3}$ $\quad A_{010} = +9.6411 \times 10^1$

$A_{110} = -1.2379 \times 10^{2}$	$A_{210} = +7.0144 \times 10^{1}$	$A_{310} = -2.4603 \times 10^{1}$
$A_{410} = +5.8737$	$A_{510} = -8.4080 \times 10^{-1}$	$A_{610} = +5.4899 \times 10^{-2}$
$A_{710} = -9.7238 \times 10^{-4}$	$A_{020} = -1.0001 \times 10^{2}$	$A_{120} = +1.0548 \times 10^{2}$
$A_{220} = -4.3144 \times 10^{1}$	$A_{320} = +1.1042 \times 10^{1}$	$A_{420} = -2.6325$
$A_{520} = +4.1179 \times 10^{-1}$	$A_{620} = -2.1584 \times 10^{-2}$	$A_{030} = +1.6540 \times 10^{1}$
$A_{130} = -1.4601$	$A_{230} = -1.0607 \times 10^{1}$	$A_{330} = 5.2353$
$A_{430} = -7.6410 \times 10^{-1}$	$A_{530} = 2.0526 \times 10^{-2}$	$A_{001} = -4.8138$
$A_{101} = 1.2272 \times 10^{1}$	$A_{201} = -9.7664$	$A_{301} = 2.9442$
$A_{401} = -8.0264 \times 10^{-2}$	$A_{501} = -1.4107 \times 10^{-1}$	$A_{601} = 2.8772 \times 10^{-2}$
$A_{701} = -1.7955 \times 10^{-3}$	$A_{011} = -4.5984 \times 10^{1}$	$A_{111} = 5.0860 \times 10^{1}$
$A_{211} = -1.8818 \times 10^{1}$	$A_{311} = 2.2897$	$A_{411} = 5.0984 \times 10^{-2}$
$A_{511} = -1.5959 \times 10^{-2}$	$A_{611} = 1.0629 \times 10^{-4}$	$A_{021} = 6.5829 \times 10^{1}$
$A_{121} = -7.4601 \times 10^{1}$	$A_{221} = 2.9075 \times 10^{1}$	$A_{321} = -4.3830$
$A_{421} = 2.1474 \times 10^{-1}$	$A_{521} = -6.2282 \times 10^{-3}$	$A_{031} = -2.0917 \times 10^{1}$
$A_{131} = 2.1692 \times 10^{1}$	$A_{231} = -7.0948$	$A_{331} = 6.2994 \times 10^{-1}$
$A_{431} = 2.6734 \times 10^{-2}$	$A_{002} = 1.4468$	$A_{102} = -3.2496$
$A_{202} = 2.7762$	$A_{302} = -1.1756$	$A_{402} = 2.6485 \times 10^{-1}$
$A_{502} = -3.0641 \times 10^{-2}$	$A_{602} = 1.4544 \times 10^{-3}$	$A_{012} = 4.2234$
$A_{112} = -4.45006$	$A_{212} = 1.4965$	$A_{312} = -1.0555 \times 10^{-1}$
$A_{412} = -2.3423 \times 10^{-2}$	$A_{512} = 2.5100 \times 10^{-3}$	$A_{022} = -6.4307$
$A_{122} = 7.2845$	$A_{222} = -2.7922$	$A_{322} = 3.9336 \times 10^{-1}$
$A_{422} = -1.2736 \times 10^{-2}$	$A_{032} = 2.1756$	$A_{132} = -2.3550$
$A_{232} = 8.2957 \times 10^{-1}$	$A_{332} = -9.3539 \times 10^{-2}$	

Table III compares the $m - \dot{\varepsilon}$ relationship predicted using Equation 9 with the experimental results [11]. (In order to keep K also in the same order of magnitude as the other variables, K was expressed in ksi; lksi = 6.8948 MPa).

In a similar manner, $K = F_2\,(X, Y, Z)$ was determined. As before, the forward selection procedure was used to select the "best fit" polynomial. Table IV presents a summary. Evidently the "best fit" polynomial in this case was given by

$$K_{\text{predicted}} = \sum_{r,s,t=0}^{r+s+t=8} A_{rst} X^r Y^s Z^t \qquad (10)$$

where

$A_{000} = -5.0970 \times 10^{4}$	$A_{100} = +5.6522 \times 10^{4}$	$A_{200} = -8.2916 \times 10^{3}$
$A_{300} = -1.4870 \times 10^{4}$	$A_{400} = +9.2913 \times 10^{3}$	$A_{500} = -2.3583 \times 10^{3}$
$A_{600} = +2.8456 \times 10^{2}$	$A_{700} = -1.3417 \times 10^{1}$	$A_{010} = -2.5015 \times 10^{5}$
$A_{110} = -3.5670 \times 10^{5}$	$A_{210} = +1.9034 \times 10^{5}$	$A_{310} = -4.2738 \times 10^{4}$
$A_{410} = +1.2639 \times 10^{3}$	$A_{510} = +1.1174 \times 10^{3}$	$A_{610} = -1.7947 \times 10^{2}$
$A_{710} = +8.1140$	$A_{020} = -3.8166 \times 10^{5}$	$A_{120} = +5.7633 \times 10^{5}$
$A_{220} = -3.4544 \times 10^{5}$	$A_{320} = +1.0375 \times 10^{5}$	$A_{420} = -1.6010 \times 10^{4}$
$A_{520} = +1.1146 \times 10^{3}$	$A_{620} = -1.9269 \times 10^{1}$	$A_{030} = +1.6300 \times 10^{5}$
$A_{130} = -2.4289 \times 10^{5}$	$A_{230} = +1.4253 \times 10^{5}$	$A_{330} = -4.1257 \times 10^{4}$
$A_{430} = +5.9146 \times 10^{3}$	$A_{530} = -3.3766 \times 10^{2}$	$A_{001} = +2.0063 \times 10^{4}$
$A_{101} = -2.7365 \times 10^{4}$	$A_{201} = +1.2442 \times 10^{4}$	$A_{301} = -8.9486 \times 10^{2}$
$A_{401} = -1.0644 \times 10^{3}$	$A_{501} = +3.8127 \times 10^{2}$	$A_{601} = -5.2111 \times 10^{1}$
$A_{701} = -2.6255$	$A_{011} = -3.5789 \times 10^{4}$	$A_{111} = +4.3646 \times 10^{4}$
$A_{211} = -1.8626 \times 10^{4}$	$A_{311} = +2.8561 \times 10^{3}$	$A_{411} = +9.0548 \times 10^{1}$
$A_{511} = -6.1769 \times 10^{1}$	$A_{611} = +4.1441$	$A_{021} = +3.0298 \times 10^{4}$
$A_{121} = -3.2779 \times 10^{4}$	$A_{221} = +1.1211 \times 10^{4}$	$A_{321} = -8.1879 \times 10^{2}$
$A_{421} = -2.1289 \times 10^{2}$	$A_{521} = +2.7234 \times 10^{1}$	$A_{031} = -1.5649 \times 10^{4}$
$A_{131} = +1.8668 \times 10^{4}$	$A_{231} = -7.9855 \times 10^{3}$	$A_{331} = +1.4312 \times 10^{3}$
$A_{431} = -8.8760 \times 10^{1}$	$A_{002} = -3.0379 \times 10^{3}$	$A_{102} = +5.0393 \times 10^{3}$
$A_{202} = -3.4024 \times 10^{3}$	$A_{302} = +1.1955 \times 10^{3}$	$A_{402} = -2.2963 \times 10^{2}$
$A_{502} = +2.2666 \times 10^{1}$	$A_{602} = -8.8053 \times 10^{-1}$	$A_{012} = +3.8793 \times 10^{3}$
$A_{112} = -5.3204 \times 10^{3}$	$A_{212} = +2.8227 \times 10^{3}$	$A_{312} = -7.3062 \times 10^{2}$
$A_{412} = +9.4356 \times 10^{1}$	$A_{512} = -5.0819$	$A_{022} = -2.0263 \times 10^{3}$
$A_{122} = +2.2057 \times 10^{3}$	$A_{222} = -7.9889 \times 10^{2}$	$A_{322} = +1.0051 \times 10^{2}$
$A_{422} = -1.5470$	$A_{032} = +7.7068 \times 10^{2}$	$A_{132} = -8.1902 \times 10^{2}$
$A_{232} = +2.8487 \times 10^{2}$	$A_{332} = -3.22946 \times 10^{1}$	

The variation of K with the experimental variables is predicted in Table V by the best fit polynomial of degree eight and compared with the experimental results.

Using the predicted values of m and K (Equations 9 and 10), the stress values corresponding to different strain rates, grain sizes and temperatures could be calculated and compared with the experimental results – Table VI.

3. Optimisation

The analysis could be further developed to optimise the deformation variables according to a pre-

TABLE III Experimental and predicted "m" values for different strain rates, grain sizes and temperatures

Temperature (T-273)	L (μm)	$\dot{\varepsilon}$ (s^{-1})	m (experimental)	m (predicted)
26	7.5 ± 0.8	3.18×10^{-5}	0.311	0.311
26	7.5 ± 0.8	7.70×10^{-5}	0.224	0.223
26	7.5 ± 0.8	1.55×10^{-4}	0.187	0.187
26	7.5 ± 0.8	2.85×10^{-4}	0.168	0.168
26	7.5 ± 0.8	6.69×10^{-4}	0.148	0.148
26	7.5 ± 0.8	1.31×10^{-3}	0.132	0.132
26	7.5 ± 0.8	2.56×10^{-3}	0.111	0.111
26	7.5 ± 0.8	6.37×10^{-3}	0.079	0.079
60	7.5 ± 0.8	3.18×10^{-5}	0.242	0.242
60	7.5 ± 0.8	7.70×10^{-5}	0.278	0.278
60	7.5 ± 0.8	1.55×10^{-4}	0.248	0.249
60	7.5 ± 0.8	2.85×10^{-4}	0.205	0.205
60	7.5 ± 0.8	6.69×10^{-4}	0.142	0.142
60	7.5 ± 0.8	1.31×10^{-3}	0.105	0.106
60	7.5 ± 0.8	2.56×10^{-3}	0.084	0.084
60	7.5 ± 0.8	6.37×10^{-3}	0.076	0.076
120	7.5 ± 0.8	3.28×10^{-5}	0.599	0.599
120	7.5 ± 0.8	7.93×10^{-5}	0.442	0.442
120	7.5 ± 0.8	1.55×10^{-4}	0.366	0.367
120	7.5 ± 0.8	3.03×10^{-4}	0.314	0.314
120	7.5 ± 0.8	7.11×10^{-4}	0.263	0.263
120	7.5 ± 0.8	1.39×10^{-3}	0.229	0.229
120	7.5 ± 0.8	2.72×10^{-3}	0.195	0.195
120	7.5 ± 0.8	6.37×10^{-3}	0.151	0.151
170	7.5 ± 0.8	3.18×10^{-5}	0.582	0.582
170	7.5 ± 0.8	7.93×10^{-5}	0.583	0.583
170	7.5 ± 0.8	1.55×10^{-4}	0.601	0.600
170	7.5 ± 0.8	2.85×10^{-4}	0.615	0.616
170	7.5 ± 0.8	7.11×10^{-4}	0.613	0.613
170	7.5 ± 0.8	1.39×10^{-3}	0.585	0.585
170	7.5 ± 0.8	2.56×10^{-3}	0.536	0.536
170	7.5 ± 0.8	9.47×10^{-3}	0.368	0.368
26	5.5 ± 0.6	3.18×10^{-5}	0.305	0.306
26	5.5 ± 0.6	7.46×10^{-5}	0.273	0.272
26	5.5 ± 0.6	1.55×10^{-4}	0.238	0.238
26	5.5 ± 0.6	3.03×10^{-4}	0.205	0.206
26	5.5 ± 0.6	7.11×10^{-4}	0.167	0.167
26	5.5 ± 0.6	1.39×10^{-3}	0.141	0.140
26	5.5 ± 0.6	2.56×10^{-3}	0.122	0.122
26	5.5 ± 0.6	5.99×10^{-3}	0.101	0.101
60	5.5 ± 0.6	3.18×10^{-5}	0.352	0.352
60	5.5 ± 0.6	8.18×10^{-5}	0.398	0.399
60	5.5 ± 0.6	1.55×10^{-4}	0.357	0.357
60	5.5 ± 0.6	3.03×10^{-4}	0.292	0.292
60	5.5 ± 0.6	7.11×10^{-4}	0.213	0.212
60	5.5 ± 0.6	1.31×10^{-3}	0.173	0.173
60	5.5 ± 0.6	2.48×10^{-3}	0.148	0.149
60	5.5 ± 0.6	5.99×10^{-3}	0.130	0.129
120	5.5 ± 0.6	3.18×10^{-5}	0.257	0.257
120	5.5 ± 0.6	7.46×10^{-5}	0.491	0.492
120	5.5 ± 0.6	1.46×10^{-4}	0.549	0.548
120	5.5 ± 0.6	2.68×10^{-4}	0.537	0.537
120	5.5 ± 0.6	6.49×10^{-4}	0.457	0.458
120	5.5 ± 0.6	1.16×10^{-3}	0.387	0.387
120	5.5 ± 0.6	2.26×10^{-3}	0.304	0.303
120	5.5 ± 0.6	4.99×10^{-3}	0.220	0.220
170	5.5 ± 0.6	7.93×10^{-5}	0.451	0.451
170	5.5 ± 0.6	1.55×10^{-4}	0.584	0.584
170	5.5 ± 0.6	2.85×10^{-4}	0.629	0.629
170	5.5 ± 0.6	7.11×10^{-4}	0.618	0.618
170	5.5 ± 0.6	1.39×10^{-3}	0.581	0.581
170	5.5 ± 0.6	2.72×10^{-3}	0.533	0.534
170	5.5 ± 0.6	6.37×10^{-3}	0.457	0.456
170	5.5 ± 0.6	1.17×10^{-2}	0.381	0.381
26	2.0 ± 0.2	2.99×10^{-5}	0.289	0.289
26	2.0 ± 0.2	7.46×10^{-5}	0.406	0.406
26	2.0 ± 0.2	1.37×10^{-4}	0.465	0.465

TABLE III (*continued*)

Temperature (T-273)	L (μm)	$\dot{\varepsilon}$ (s^{-1})	m (experimental)	m (predicted)
26	2.0 ± 0.2	2.77×10^{-4}	0.506	0.506
26	2.0 ± 0.2	6.29×10^{-4}	0.513	0.512
26	2.0 ± 0.2	1.31×10^{-3}	0.482	0.482
26	2.0 ± 0.2	2.33×10^{-3}	0.438	0.439
26	2.0 ± 0.2	4.70×10^{-3}	0.371	0.371
68	2.0 ± 0.2	3.18×10^{-5}	0.365	0.365
68	2.0 ± 0.2	7.70×10^{-5}	0.427	0.426
68	2.0 ± 0.2	1.46×10^{-4}	0.460	0.460
68	2.0 ± 0.2	2.85×10^{-4}	0.486	0.487
68	2.0 ± 0.2	6.90×10^{-4}	0.507	0.507
68	2.0 ± 0.2	1.35×10^{-3}	0.511	0.511
68	2.0 ± 0.2	2.48×10^{-3}	0.505	0.504
68	2.0 ± 0.2	5.64×10^{-3}	0.479	0.479
90	2.0 ± 0.2	3.18×10^{-5}	0.443	0.442
90	2.0 ± 0.2	7.70×10^{-5}	0.434	0.435
90	2.0 ± 0.2	1.46×10^{-4}	0.446	0.446
90	2.0 ± 0.2	2.85×10^{-4}	0.467	0.466
90	2.0 ± 0.2	6.90×10^{-4}	0.498	0.497
90	2.0 ± 0.2	1.35×10^{-3}	0.516	0.517
90	2.0 ± 0.2	2.48×10^{-3}	0.523	0.524
90	2.0 ± 0.2	5.47×10^{-3}	0.513	0.513
120	2.0 ± 0.2	3.09×10^{-5}	0.445	0.445
120	2.0 ± 0.2	7.46×10^{-5}	0.511	0.511
120	2.0 ± 0.2	1.46×10^{-4}	0.523	0.523
120	2.0 ± 0.2	2.68×10^{-4}	0.526	0.526
120	2.0 ± 0.2	6.69×10^{-4}	0.534	0.534
120	2.0 ± 0.2	1.23×10^{-3}	0.548	0.548
120	2.0 ± 0.2	2.40×10^{-3}	0.571	0.571
120	2.0 ± 0.2	5.47×10^{-3}	0.592	0.592

determined criterion, e.g. minimisation of power consumption. This was achieved in the following manner.

Power consumption per unit volume of material deformed, P, is given by

$$P_{\text{experimental}} = \sigma_{\text{experimental}} \cdot \dot{\varepsilon}_{\text{experimental}} \quad (11)$$

on the other hand,

$$P_{\text{predicted}} = K_{\text{predicted}} \cdot \dot{\varepsilon}^{(m_{\text{predicted}}+1)}$$

(using Equations 1 and 11)

Table VII shows the variation with strain rate of power consumption per unit volume of material deformed for different combinations of grain size and temperature.

TABLE IV Selection of "best fit" polynomial for predicting the values of K

Degree of equation	Root mean square error (%)	Residual sum of squares	$\frac{(Er_n - Er_{n+1}) \cdot 100}{Er_n}$ (%)
1	113.47	7.769×10^4	–
2	100.36	6.078×10^4	21.77
3	89.54	4.838×10^4	20.40
4	74.53	3.352×10^4	30.72
5	52.39	1.657×10^4	50.57
6	27.75	4.646×10^3	71.96
7	13.13	1.040×10^3	77.62
8	3.72	8.365×10^1	91.96
9	2.82	4.807×10^1	42.53
10	9.39	5.329×10^2	−1008.59
11	2.08	2.622×10^1	95.08
12	2.06	2.565×10^1	2.17

From Table VII it is evident that for each combination of grain size and temperature there is a range of strain rate values for which the power consumption is quite low. Thus, one could select a convenient combination of temperature, grain size and strain rate range to complete the deformation with minimum power consumption and the least possible forming time.

Moreover, in accordance with the experimental results the predicted power consumption decreased with decreasing grain size and/or increasing temperature (Table VII).

The strength of the above analysis lies in its ability to allow for internal variations in strain rate, m and K, while at the same time permitting the computation of the external load and power consumption for possible optimisation.

Finally, although this numerical procedure has been validated by considering superplastic deformation, it can be used for all situations where a particular parameter is influenced by a number of independent variables.

4. Conclusions

An earlier multi-dimensional regression analysis [9] could be extended to four dimensional space where the strain-rate sensitivity index, m, and the strength parameter, K, are expressible as functions of strain rate, grain size and temperature. These expressions were useful for predicting the external load and power consumption so that an optimisation of the operating conditions (for minimum power consumption) became

TABLE V Experimental and predicted K values for varying strain rates, grain sizes and temperatures. (K values are in units of ksi; 1 ksi = 6.8948 MPa)

Temperature (T-273)	L (μm)	$\dot{\varepsilon}$ (s^{-1})	K (experimental)	K (predicted)
26	7.5 ± 0.8	3.18×10^{-5}	69.72	69.83
26	7.5 ± 0.8	7.70×10^{-5}	29.50	29.02
26	7.5 ± 0.8	1.55×10^{-4}	20.75	21.23
26	7.5 ± 0.8	2.85×10^{-4}	17.92	18.21
26	7.5 ± 0.8	6.69×10^{-4}	15.17	14.99
26	7.5 ± 0.8	1.31×10^{-3}	13.88	12.95
26	7.5 ± 0.8	2.56×10^{-3}	11.96	12.93
26	7.5 ± 0.8	6.37×10^{-3}	10.06	9.85
60	7.5 ± 0.8	3.18×10^{-5}	22.33	22.40
60	7.5 ± 0.8	7.70×10^{-5}	32.40	32.29
60	7.5 ± 0.8	1.55×10^{-4}	24.66	24.82
60	7.5 ± 0.8	2.85×10^{-4}	16.76	16.43
60	7.5 ± 0.8	6.69×10^{-4}	10.69	10.76
60	7.5 ± 0.8	1.31×10^{-3}	8.08	8.87
60	7.5 ± 0.8	2.56×10^{-3}	7.05	6.26
60	7.5 ± 0.8	6.37×10^{-3}	6.65	6.84
120	7.5 ± 0.8	3.28×10^{-5}	277.84	277.39
120	7.5 ± 0.8	7.93×10^{-5}	56.90	59.07
120	7.5 ± 0.8	1.55×10^{-4}	29.55	26.61
120	7.5 ± 0.8	3.03×10^{-4}	19.21	20.07
120	7.5 ± 0.8	7.11×10^{-4}	12.27	13.64
120	7.5 ± 0.8	1.39×10^{-3}	9.84	8.38
120	7.5 ± 0.8	2.72×10^{-3}	8.29	8.82
120	7.5 ± 0.8	6.37×10^{-3}	6.36	6.31
170	7.5 ± 0.8	3.18×10^{-5}	23.34	23.52
170	7.5 ± 0.8	7.93×10^{-5}	22.50	21.64
170	7.5 ± 0.8	1.55×10^{-4}	27.35	28.71
170	7.5 ± 0.8	2.85×10^{-4}	32.62	31.75
170	7.5 ± 0.8	7.11×10^{-4}	29.83	30.53
170	7.5 ± 0.8	1.39×10^{-3}	25.14	24.26
170	7.5 ± 0.8	2.56×10^{-3}	17.89	18.35
170	7.5 ± 0.8	9.47×10^{-3}	7.90	7.87
26	5.5 ± 0.6	3.18×10^{-5}	57.21	57.06
26	5.5 ± 0.6	7.46×10^{-5}	41.18	41.84
26	5.5 ± 0.6	1.55×10^{-4}	29.80	29.48
26	5.5 ± 0.6	3.03×10^{-4}	23.34	22.24
26	5.5 ± 0.6	7.11×10^{-4}	16.72	17.94
26	5.5 ± 0.6	1.39×10^{-3}	14.25	14.85
26	5.5 ± 0.6	2.56×10^{-3}	12.39	11.21
26	5.5 ± 0.6	5.99×10^{-3}	11.31	11.62
60	5.5 ± 0.6	3.18×10^{-5}	44.74	44.82
60	5.5 ± 0.6	8.18×10^{-5}	71.14	70.22
60	5.5 ± 0.6	1.55×10^{-4}	49.18	50.91
60	5.5 ± 0.6	3.03×10^{-4}	29.06	28.54
60	5.5 ± 0.6	7.11×10^{-4}	15.29	14.31
60	5.5 ± 0.6	1.31×10^{-3}	11.63	12.01
60	5.5 ± 0.6	2.48×10^{-3}	10.15	10.58
60	5.5 ± 0.6	5.99×10^{-3}	9.13	8.96
120	5.5 ± 0.6	3.18×10^{-5}	2.87	3.15
120	5.5 ± 0.6	7.46×10^{-5}	30.87	30.21
120	5.5 ± 0.6	1.46×10^{-4}	50.09	49.95
120	5.5 ± 0.6	2.68×10^{-4}	46.82	47.61
120	5.5 ± 0.6	6.49×10^{-4}	24.72	25.67
120	5.5 ± 0.6	1.16×10^{-3}	15.93	13.67
120	5.5 ± 0.6	2.26×10^{-3}	8.92	10.22
120	5.5 ± 0.6	4.99×10^{-3}	5.38	5.16
170	5.5 ± 0.6	7.93×10^{-5}	5.08	4.29
170	5.5 ± 0.6	1.55×10^{-4}	17.40	20.40
170	5.5 ± 0.6	2.85×10^{-4}	25.26	21.96
170	5.5 ± 0.6	7.11×10^{-4}	22.67	23.47
170	5.5 ± 0.6	1.39×10^{-3}	19.05	20.18
170	5.5 ± 0.6	2.72×10^{-3}	13.16	12.13
170	5.5 ± 0.6	6.37×10^{-3}	8.68	8.89
170	5.5 ± 0.6	1.17×10^{-2}	6.34	6.33
26	2.0 ± 0.2	2.99×10^{-5}	7.96	7.99
26	2.0 ± 0.2	7.46×10^{-5}	25.07	24.94

TABLE V (*continued*)

Temperature (T-273)	L (μm)	$\dot{\varepsilon}$ (s^{-1})	K (experimental)	K (predicted)
26	2.0 ± 0.2	1.37×10^{-4}	44.87	44.81
26	2.0 ± 0.2	2.77×10^{-4}	61.43	62.15
26	2.0 ± 0.2	6.29×10^{-4}	65.22	64.28
26	2.0 ± 0.2	1.31×10^{-3}	52.85	53.19
26	2.0 ± 0.2	2.33×10^{-3}	41.59	41.75
26	2.0 ± 0.2	4.70×10^{-3}	27.19	27.10
68	2.0 ± 0.2	3.18×10^{-5}	7.73	7.89
68	2.0 ± 0.2	7.70×10^{-5}	14.51	14.39
68	2.0 ± 0.2	1.46×10^{-4}	18.93	18.58
68	2.0 ± 0.2	2.85×10^{-4}	24.78	24.67
68	2.0 ± 0.2	6.90×10^{-4}	28.69	29.34
68	2.0 ± 0.2	1.35×10^{-3}	28.50	29.15
68	2.0 ± 0.2	2.48×10^{-3}	29.04	27.78
68	2.0 ± 0.2	5.64×10^{-3}	24.14	24.52
90	2.0 ± 0.2	3.18×10^{-5}	11.30	11.04
90	2.0 ± 0.2	7.70×10^{-5}	10.14	10.56
90	2.0 ± 0.2	1.46×10^{-4}	11.56	11.67
90	2.0 ± 0.2	2.85×10^{-4}	13.83	14.17
90	2.0 ± 0.2	6.90×10^{-4}	17.57	16.42
90	2.0 ± 0.2	1.35×10^{-3}	19.23	18.83
90	2.0 ± 0.2	2.48×10^{-3}	21.13	22.60
90	2.0 ± 0.2	5.47×10^{-3}	19.09	18.59
120	2.0 ± 0.2	3.09×10^{-5}	5.64	5.65
120	2.0 ± 0.2	7.46×10^{-5}	10.93	11.39
120	2.0 ± 0.2	1.46×10^{-4}	12.47	11.33
120	2.0 ± 0.2	2.68×10^{-4}	12.51	12.86
120	2.0 ± 0.2	6.69×10^{-4}	12.59	13.91
120	2.0 ± 0.2	1.23×10^{-3}	15.37	14.20
120	2.0 ± 0.2	2.40×10^{-3}	17.54	17.62
120	2.0 ± 0.2	5.47×10^{-3}	18.79	18.89

TABLE VI Experimental and predicted stress values for varying strain rates, grain sizes and temperatures

Temperature (T-273)	L (μm)	$\dot{\varepsilon}$ (s^{-1})	σ (psi experimental)	σ (psi predicted)
26	7.5 ± 0.8	3.18×10^{-5}	2780	2786
26	7.5 ± 0.8	7.70×10^{-5}	3550	3477
26	7.5 ± 0.8	1.55×10^{-4}	4005	4117
26	7.5 ± 0.8	2.85×10^{-4}	4530	3139
26	7.5 ± 0.8	6.69×10^{-4}	5110	5082
26	7.5 ± 0.8	1.31×10^{-3}	5780	5392
26	7.5 ± 0.8	2.56×10^{-3}	6140	6667
26	7.5 ± 0.8	6.37×10^{-3}	6730	6606
60	7.5 ± 0.8	3.18×10^{-5}	1810	1828
60	7.5 ± 0.8	7.70×10^{-5}	2310	2322
60	7.5 ± 0.8	1.55×10^{-4}	2780	2795
60	7.5 ± 0.8	2.85×10^{-4}	3140	3082
60	7.5 ± 0.8	6.69×10^{-4}	3770	3811
60	7.5 ± 0.8	1.31×10^{-3}	4005	4389
60	7.5 ± 0.8	2.56×10^{-3}	4260	3792
60	7.5 ± 0.8	6.37×10^{-3}	4530	4659
120	7.5 ± 0.8	3.28×10^{-5}	570	583
120	7.5 ± 0.8	7.93×10^{-5}	870	910
120	7.5 ± 0.8	1.55×10^{-4}	1180	1064
120	7.5 ± 0.8	3.03×10^{-4}	1510	1593
120	7.5 ± 0.8	7.11×10^{-4}	1810	2027
120	7.5 ± 0.8	1.39×10^{-3}	2180	1858
120	7.5 ± 0.8	2.72×10^{-3}	2610	2787
120	7.5 ± 0.8	6.37×10^{-3}	2950	2941
170	7.5 ± 0.8	3.18×10^{-5}	60	56
170	7.5 ± 0.8	7.93×10^{-5}	90	89
170	7.5 ± 0.8	1.55×10^{-4}	140	149
170	7.5 ± 0.8	2.85×10^{-4}	210	206
170	7.5 ± 0.8	7.11×10^{-4}	350	360
170	7.5 ± 0.8	1.39×10^{-3}	540	517

TABLE VI (*continued*)

Temperature (T-273)	L (μm)	$\dot{\varepsilon}$ (s^{-1})	σ (psi experimental)	σ (psi predicted)
170	7.5 ± 0.8	2.56×10^{-3}	730	749
170	7.5 ± 0.8	9.47×10^{-3}	1420	1417
26	5.5 ± 0.6	3.18×10^{-5}	2410	2397
26	5.5 ± 0.6	7.46×10^{-5}	3070	3155
26	5.5 ± 0.6	1.55×10^{-4}	3680	3656
26	5.5 ± 0.6	3.03×10^{-4}	4410	4190
26	5.5 ± 0.6	7.11×10^{-4}	4980	5346
26	5.5 ± 0.6	1.39×10^{-3}	5620	5912
26	5.5 ± 0.6	2.56×10^{-3}	5970	5412
26	5.5 ± 0.6	5.99×10^{-3}	6740	6930
60	5.5 ± 0.6	3.18×10^{-5}	1160	1170
60	5.5 ± 0.6	8.18×10^{-5}	1670	1643
60	5.5 ± 0.6	1.55×10^{-4}	2130	2220
60	5.5 ± 0.6	3.03×10^{-4}	2720	2680
60	5.5 ± 0.6	7.11×10^{-4}	3260	3078
60	5.5 ± 0.6	1.31×10^{-3}	3680	3810
60	5.5 ± 0.6	2.48×10^{-3}	4150	4327
60	5.5 ± 0.6	5.99×10^{-3}	4690	4631
120	5.5 ± 0.6	3.18×10^{-5}	200	220
120	5.5 ± 0.6	7.46×10^{-5}	290	281
120	5.5 ± 0.6	1.46×10^{-4}	390	395
120	5.5 ± 0.6	2.68×10^{-4}	560	576
120	5.5 ± 0.6	6.49×10^{-4}	860	891
120	5.5 ± 0.6	1.16×10^{-3}	1160	999
120	5.5 ± 0.6	2.26×10^{-3}	1400	1614
120	5.5 ± 0.6	4.99×10^{-3}	1670	1608
170	5.5 ± 0.6	7.93×10^{-5}	70	61
170	5.5 ± 0.6	1.55×10^{-4}	100	122
170	5.5 ± 0.6	2.85×10^{-4}	150	130
170	5.5 ± 0.6	7.11×10^{-4}	260	265
170	5.5 ± 0.6	1.39×10^{-3}	420	442
170	5.5 ± 0.6	2.72×10^{-3}	560	518
170	5.5 ± 0.6	6.37×10^{-3}	860	886
170	5.5 ± 0.6	1.17×10^{-2}	1160	1162
26	2.0 ± 0.2	2.99×10^{-5}	390	394
26	2.0 ± 0.2	7.46×10^{-5}	530	526
26	2.0 ± 0.2	1.37×10^{-4}	710	717
26	2.0 ± 0.2	2.77×10^{-4}	970	982
26	2.0 ± 0.2	6.29×10^{-4}	1490	1478
26	2.0 ± 0.2	1.31×10^{-3}	2140	2170
26	2.0 ± 0.2	2.33×10^{-3}	2910	2918
26	2.0 ± 0.2	4.70×10^{-3}	3710	3710
68	2.0 ± 0.2	3.18×10^{-5}	180	180
68	2.0 ± 0.2	7.70×10^{-5}	250	255
68	2.0 ± 0.2	1.46×10^{-4}	320	320
68	2.0 ± 0.2	2.85×10^{-4}	470	464
68	2.0 ± 0.2	6.90×10^{-4}	710	734
68	2.0 ± 0.2	1.35×10^{-3}	970	997
68	2.0 ± 0.2	2.48×10^{-3}	1400	1350
68	2.0 ± 0.2	5.64×10^{-3}	2020	2052
90	2.0 ± 0.2	3.18×10^{-5}	110	114
90	2.0 ± 0.2	7.70×10^{-5}	170	171
90	2.0 ± 0.2	1.46×10^{-4}	220	228
90	2.0 ± 0.2	2.85×10^{-4}	300	316
90	2.0 ± 0.2	6.90×10^{-4}	470	440
90	2.0 ± 0.2	1.35×10^{-3}	630	618
90	2.0 ± 0.2	2.48×10^{-3}	910	974
90	2.0 ± 0.2	5.47×10^{-3}	1320	1285
120	2.0 ± 0.2	3.09×10^{-5}	55	55
120	2.0 ± 0.2	7.46×10^{-5}	85	89
120	2.0 ± 0.2	1.46×10^{-4}	120	112
120	2.0 ± 0.2	2.68×10^{-4}	170	170
120	2.0 ± 0.2	6.69×10^{-4}	250	281
120	2.0 ± 0.2	1.23×10^{-3}	390	361
120	2.0 ± 0.2	2.40×10^{-3}	560	562
120	2.0 ± 0.2	5.47×10^{-3}	860	865

Residual sum of squares = 3.79×10^6; RMS error = 10.68%.

TABLE VII A comparison between the experimental and predicted power consumption per unit volume of material deformed corresponding to different strain rates, grain sizes and temperatures. (Units of power are ksi per second per cubic inch)

Temperature (T-273)	L (μm)	$\dot{\varepsilon}$ (s^{-1})	Power (experimental)	Power (predicted)
26	7.5 ± 0.8	3.18×10^{-5}	0.088	0.089
26	7.5 ± 0.8	7.70×10^{-5}	0.273	0.268
26	7.5 ± 0.8	1.55×10^{-4}	0.621	0.638
26	7.5 ± 0.8	2.85×10^{-4}	1.291	0.895
26	7.5 ± 0.8	6.69×10^{-4}	3.419	3.400
26	7.5 ± 0.8	1.31×10^{-3}	7.572	7.064
26	7.5 ± 0.8	2.56×10^{-3}	15.719	17.067
26	7.5 ± 0.8	6.37×10^{-3}	42.870	42.083
60	7.5 ± 0.8	3.18×10^{-5}	0.058	0.058
60	7.5 ± 0.8	7.70×10^{-5}	0.178	0.179
60	7.5 ± 0.8	1.55×10^{-4}	0.431	0.433
60	7.5 ± 0.8	2.85×10^{-4}	0.895	0.879
60	7.5 ± 0.8	6.69×10^{-4}	2.522	2.550
60	7.5 ± 0.8	1.31×10^{-3}	5.247	5.750
60	7.5 ± 0.8	2.56×10^{-3}	10.906	9.707
60	7.5 ± 0.8	6.37×10^{-3}	28.856	29.680
120	7.5 ± 0.8	3.28×10^{-5}	0.019	0.019
120	7.5 ± 0.8	7.93×10^{-5}	0.069	0.072
120	7.5 ± 0.8	1.55×10^{-4}	0.183	0.165
120	7.5 ± 0.8	3.03×10^{-4}	0.458	0.483
120	7.5 ± 0.8	7.11×10^{-4}	1.287	1.441
120	7.5 ± 0.8	1.39×10^{-3}	3.030	2.582
120	7.5 ± 0.8	2.72×10^{-3}	7.099	7.581
120	7.5 ± 0.8	6.37×10^{-3}	18.792	18.731
170	7.5 ± 0.8	3.18×10^{-5}	0.002	0.002
170	7.5 ± 0.8	7.93×10^{-5}	0.007	0.007
170	7.5 ± 0.8	1.55×10^{-4}	0.022	0.023
170	7.5 ± 0.8	2.85×10^{-4}	0.060	0.059
170	7.5 ± 0.8	7.11×10^{-4}	0.249	0.256
170	7.5 ± 0.8	1.39×10^{-3}	0.751	0.718
170	7.5 ± 0.8	2.56×10^{-3}	1.869	1.917
170	7.5 ± 0.8	9.47×10^{-3}	13.447	13.415
26	5.5 ± 0.6	3.18×10^{-5}	0.077	0.076
26	5.5 ± 0.6	7.46×10^{-5}	0.023	0.235
26	5.5 ± 0.6	1.55×10^{-4}	0.570	0.567
26	5.5 ± 0.6	3.03×10^{-4}	1.336	1.270
26	5.5 ± 0.6	7.11×10^{-4}	3.541	3.801
26	5.5 ± 0.6	1.39×10^{-3}	7.812	8.217
26	5.5 ± 0.6	2.56×10^{-3}	15.283	13.855
26	5.5 ± 0.6	5.99×10^{-3}	40.373	41.512
60	5.5 ± 0.6	3.18×10^{-5}	0.037	0.037
60	5.5 ± 0.6	8.18×10^{-5}	0.137	0.134
60	5.5 ± 0.6	1.55×10^{-4}	0.330	0.344
60	5.5 ± 0.6	3.03×10^{-4}	0.824	0.812
60	5.5 ± 0.6	7.11×10^{-4}	2.318	2.189
60	5.5 ± 0.6	1.31×10^{-3}	4.821	4.991
60	5.5 ± 0.6	2.48×10^{-3}	10.292	10.732
60	5.5 ± 0.6	5.99×10^{-3}	28.093	27.737
120	5.5 ± 0.6	3.18×10^{-5}	0.006	0.007
120	5.5 ± 0.6	7.46×10^{-5}	0.022	0.021
120	5.5 ± 0.6	1.46×10^{-4}	0.057	0.058
120	5.5 ± 0.6	2.68×10^{-4}	0.150	0.154
120	5.5 ± 0.6	6.49×10^{-4}	0.558	0.578
120	5.5 ± 0.6	1.16×10^{-3}	1.346	1.159
120	5.5 ± 0.6	2.26×10^{-3}	3.164	3.647
120	5.5 ± 0.6	4.99×10^{-3}	8.333	8.023
170	5.5 ± 0.6	7.93×10^{-5}	0.006	0.005
170	5.5 ± 0.6	1.55×10^{-4}	0.016	0.019
170	5.5 ± 0.6	2.85×10^{-4}	0.043	0.037
170	5.5 ± 0.6	7.11×10^{-4}	0.185	0.189
170	5.5 ± 0.6	1.39×10^{-3}	0.584	0.614
170	5.5 ± 0.6	2.72×10^{-3}	1.523	1.409
170	5.5 ± 0.6	6.37×10^{-3}	5.478	5.646
170	5.5 ± 0.6	1.17×10^{-2}	13.572	13.598
26	2.0 ± 0.2	2.99×10^{-5}	0.012	0.012
26	2.0 ± 0.2	7.46×10^{-5}	0.040	0.039

TABLE VII (*continued*)

Temperature (T-273)	L (μm)	$\dot{\varepsilon}$ (s^{-1})	Power (experimental)	Power (predicted)
26	2.0 ± 0.2	1.37×10^{-4}	0.097	0.098
26	2.0 ± 0.2	2.77×10^{-4}	0.269	0.272
26	2.0 ± 0.2	6.29×10^{-4}	0.937	0.930
26	2.0 ± 0.2	1.31×10^{-3}	2.803	2.843
26	2.0 ± 0.2	2.33×10^{-3}	6.780	6.800
26	2.0 ± 0.2	4.70×10^{-3}	17.437	17.437
68	2.0 ± 0.2	3.18×10^{-5}	0.006	0.006
68	2.0 ± 0.2	7.70×10^{-5}	0.019	0.020
68	2.0 ± 0.2	1.46×10^{-4}	0.047	0.047
68	2.0 ± 0.2	2.85×10^{-4}	0.134	0.132
68	2.0 ± 0.2	6.90×10^{-4}	0.490	0.506
68	2.0 ± 0.2	1.35×10^{-3}	1.310	1.346
68	2.0 ± 0.2	2.48×10^{-3}	3.472	3.348
68	2.0 ± 0.2	5.64×10^{-3}	11.393	11.575
90	2.0 ± 0.2	3.18×10^{-5}	0.004	0.004
90	2.0 ± 0.2	7.70×10^{-5}	0.013	0.013
90	2.0 ± 0.2	1.46×10^{-4}	0.003	0.003
90	2.0 ± 0.2	2.85×10^{-4}	0.086	0.090
90	2.0 ± 0.2	6.90×10^{-4}	0.324	0.304
90	2.0 ± 0.2	1.35×10^{-3}	0.851	0.834
90	2.0 ± 0.2	2.48×10^{-3}	2.257	2.416
90	2.0 ± 0.2	5.47×10^{-3}	7.220	7.027
120	2.0 ± 0.2	3.09×10^{-5}	0.002	0.002
120	2.0 ± 0.2	7.46×10^{-5}	0.006	0.007
120	2.0 ± 0.2	1.46×10^{-4}	0.018	0.016
120	2.0 ± 0.2	2.68×10^{-4}	0.046	0.046
120	2.0 ± 0.2	6.69×10^{-4}	0.167	0.188
120	2.0 ± 0.2	1.23×10^{-3}	0.480	0.444
120	2.0 ± 0.2	2.40×10^{-3}	1.344	1.349
120	2.0 ± 0.2	5.47×10^{-3}	4.704	4.733

Residual sum of squares = 10.39; RMS error = 8.26%

possible. The technique was validated by analysing the experimental data on the tin–lead eutectic alloy generated by Zehr and Backofen [11]. It is also concluded that the technique could be useful for all problems in which a particular parameter depends on a number of independent variables.

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